130217 Aerodynamics exam

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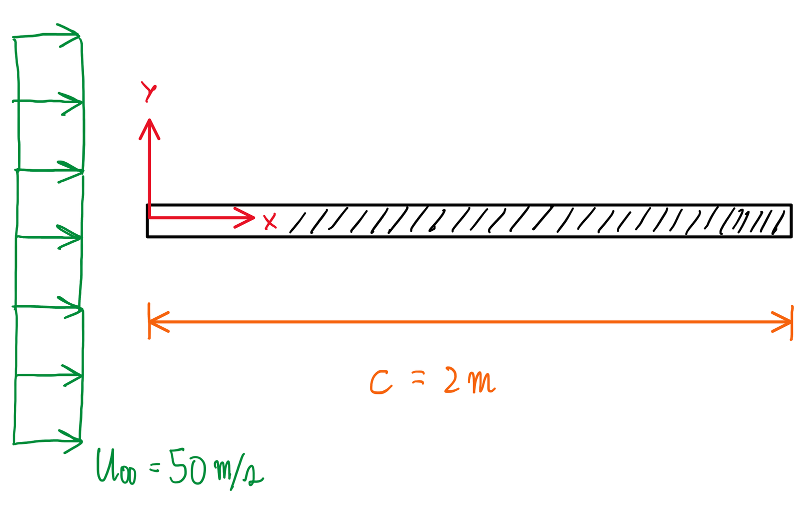
Considering an airflow which $U_\infty = 50\,m/s$ and has an imersed flat plate exhibiting a stall condition. The reference system has x and y are parallel and vertical to the flat plate, respectively. In addition, the origin of the reference system coincide with the leading edge of the flat plate. There are two another coordinate frames, 1 and 2. The first establishes that x_1 has a flow attached and with a profile of:

$$u_1(\tilde{y}) = U_\infty\left[(\tilde{y}-1)^3+1\right] \quad , \quad \tilde{y} = \frac{y}{\delta(x)}$$



Where $\delta(x)$ is the boundary layer thickness. The first part of this exercise has the following questions. 1 - Calculate the Reynolds number:

$$Re = \frac{U_\infty \cdot x}{\nu} = \frac{50\cdot 2}{15\cdot 10^{-6}} = 6666666.667 = 6{,}667\cdot 10^6 \longrightarrow \text{TURBULENT}$$

2 - Calculate the viscous stress at the wall in X_1:

$$\tau_{12} \approx \mu\cdot\frac{\partial u}{\partial y}$$

$$u_1(\tilde{y}) = U_\infty\cdot\left[(\tilde{y}-1)^3+1\right] = U_\infty\left[(\tilde{y}^2-2\tilde{y}+1)(\tilde{y}-1)+1\right]$$

$$u_1(\tilde{y}) = U_\infty\left[\tilde{y}^3-\tilde{y}^2-2\tilde{y}^2+2\tilde{y}+\tilde{y}-1+1\right] = U_\infty\left[\tilde{y}^3-3\tilde{y}^2+3\tilde{y}\right]$$

$$u_1(\tilde{y}) = U_\infty\left(\frac{y^3}{\delta(x)^3} - 3\cdot\frac{y^2}{\delta(x)^2} + 3\cdot\frac{y}{\delta(x)}\right)$$

$$\frac{\partial u_1}{\partial y} = U_\infty\left(3\frac{y^2}{\delta(x)^3} - 3\cdot 2\frac{y}{\delta(x)^2} + 3\frac{1}{\delta(x)}\right) = U_\infty\left(3\frac{y^2}{\delta(x)^3} - 6\frac{y}{\delta(x)^2} + 3\frac{1}{\delta(x)}\right)$$

$$\tau_w = \mu\cdot\frac{\partial u}{\partial y} = \mu\cdot U_\infty\left(3\cdot\cancelto{0}{\frac{y^2}{\delta(x)^3}} - 6\cancelto{0}{\frac{y}{\delta(x)^2}} + 3\frac{1}{\delta(x)}\right) = \frac{3\mu\cdot U_\infty}{\delta(x)}$$

$$\tau_w = 3\cdot\frac{\mu\cdot U_\infty}{\delta(x)}$$

$$\delta(x) = \frac{0.37\cdot x}{Re^{1/5}} = 0.37\cdot\frac{x}{\left(\frac{U_\infty\cdot x}{\nu}\right)^{1/5}} = 0.37\cdot\frac{x\cdot\nu^{1/5}}{U_\infty^{1/5}\cdot x^{1/5}} = 0.37\cdot\frac{x^{4/5}\cdot\nu^{1/5}}{U_\infty^{1/5}}$$

$$\tau_w = 3\cdot\frac{\mu\cdot U_\infty}{0.37\cdot\frac{x^{4/5}\cdot\nu^{1/5}}{U_\infty^{1/5}}} = 3\cdot\frac{\mu\cdot U_\infty\cdot U_\infty^{1/5}}{0.37\cdot x^{4/5}\cdot\nu^{1/5}} = \frac{3}{0.37}\cdot\frac{\mu\cdot U_\infty^{6/5}}{x^{4/5}\cdot\nu^{1/5}}$$

$$\tau_w = \frac{3}{0.37}\cdot\mu\cdot\sqrt[5]{\frac{U_\infty^6}{x^4\cdot\nu}} = \frac{3}{0.37}\cdot 18\cdot 10^{-6}\cdot\sqrt[5]{\frac{50^6}{2^4\cdot 15\cdot 10^{-6}}} = 0.0845\,Pa$$

The second part of this exercise deals with second reference coordinate X_2, which the velocity profile is given by:

$$u_2(\tilde{y}) = U_\infty\left[2\tilde{y}^4-5\tilde{y}^3+3\tilde{y}^2+\tilde{y}\right]$$

3 - Define what motivates separation: The condition for flow separation is indicated by the velocity profile curvature, because for conditions at the wall, the velocity profile has the same sign of the pressure gradient. For the favorable pressure gradient, the curvature is negative throughout the surface and no separation occurs. Hence, to occur separation, the pressure gradient should be positive. When this occurs, the curvature changes sign and separation occurs for $\frac{\partial u}{\partial y} \leqslant 0$. Hence, the conditions for separation are $\frac{\partial u}{\partial y} \leqslant 0$ and $\frac{\partial P}{\partial x} > 0$.

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_w \approx \frac{1}{\mu}\cdot\frac{\partial P}{\partial x} \quad ; \quad U_\infty\left[2\cdot\frac{y^4}{\delta(x)^4} - 5\cdot\frac{y^3}{\delta(x)^3} + 3\frac{y^2}{\delta(x)^2} + \frac{y}{\delta(x)}\right]$$

$$\frac{\partial u}{\partial y} = U_\infty\left[2\cdot 4\frac{y^3}{\delta(x)^4} - 5\cdot 3\frac{y^2}{\delta(x)^3} + 3\cdot 2\frac{y}{\delta(x)^2} + \frac{1}{\delta(x)}\right]$$

$$\frac{\partial u}{\partial y} = U_\infty\left[8\cdot\cancelto{0}{\frac{y^3}{\delta(x)^4}} - 15\cancelto{0}{\frac{y^2}{\delta(x)^3}} + 6\cdot\cancelto{0}{\frac{y}{\delta(x)^2}} + \frac{1}{\delta(x)}\right] = \frac{U_\infty}{\delta(x)}$$

$$\frac{\partial^2 u}{\partial y^2} = U_\infty\left[8\cdot 3\cdot\frac{y^2}{\delta(x)^4} - 15\cdot 2\frac{y}{\delta(x)^3} + 6\frac{1}{\delta(x)^2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = U_\infty\left[24\cancelto{0}{\frac{y^2}{\delta(x)^4}} - 30\cdot\cancelto{0}{\frac{y}{\delta(x)^3}} + 6\frac{1}{\delta(x)^2}\right] = 6\cdot\frac{U_\infty}{\delta(x)^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\cdot\frac{\partial P}{\partial x} = 6\frac{U_\infty}{\delta(x)^2}$$

$$\frac{\partial P}{\partial x} = 6\frac{\mu\cdot U_\infty}{\delta(x)^2}$$

$$\delta(x) = 0.37\cdot\frac{x}{Re^{1/5}} = 0.37\cdot\frac{2}{6666666.667^{1/5}} = 0.0319\,m$$

$$\frac{\partial P}{\partial x} = 6\cdot\frac{18\cdot 10^{-6}\cdot 50}{0.0319^2} = 5.307 \longrightarrow \text{Positive}$$

Hence, the curvature is positive, which confirms that do not occur separation, but $\frac{\partial P}{\partial x}$ is also positive. This means that the pressure gradient adverse, but without separation.

There are instabilities on the flow, but not enough to trigger separation. The next part of the exercise proposes a third coordinate, which is x_3 that has the following velocity profile:

$$u_3(\tilde{y}) = U_\infty\cdot\left[4\tilde{y}^4 - 11\tilde{y}^3 + 9\tilde{y}^2 - \tilde{y}\right]$$

5 - Calculate the viscous stress at the wall

$$u_3\left(\frac{y}{\delta(x)}\right) = U_\infty\left[4\cdot\frac{y^4}{\delta(x)^4} - 11\frac{y^3}{\delta(x)^3} + 9\frac{y^2}{\delta(x)^2} - \frac{y}{\delta(x)}\right]$$

$$\frac{\partial u}{\partial y} = U_\infty\left[4\cdot 4\cdot\frac{y^3}{\delta(x)^4} - 11\cdot 3\frac{y^2}{\delta(x)^3} + 9\cdot 2\frac{y}{\delta(x)^2} - \frac{1}{\delta(x)}\right]$$

$$\frac{\partial u}{\partial y} = U_\infty\left[16\frac{y^3}{\delta(x)^4} - 33\frac{y^2}{\delta(x)^3} + 18\frac{y}{\delta(x)^2} - \frac{1}{\delta(x)}\right]$$

$$\tau_w = \mu\cdot\frac{\partial u}{\partial y} = \mu\cdot U_\infty\left[16\cdot\cancelto{0}{\frac{y^3}{\delta(x)^4}} - 33\cancelto{0}{\frac{y^2}{\delta(x)^3}} + \cancelto{0}{18\frac{y}{\delta(x)^2}} - \frac{1}{\delta(x)}\right]$$

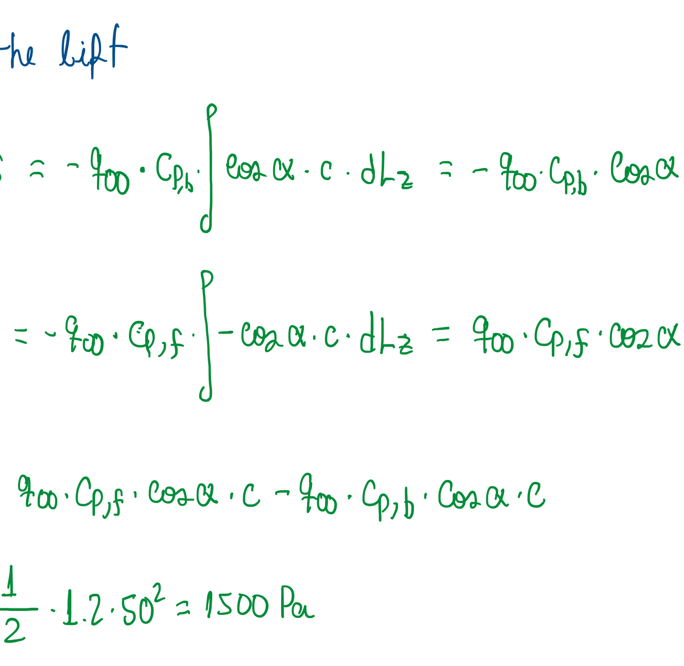
$$\tau_w = \mu\cdot U_\infty\left(-\frac{1}{\delta(x)}\right) = -\frac{\mu\cdot U_\infty}{\delta(x)} = -18\cdot 10^{-6}\cdot 50\cdot\frac{1}{\delta(x)}$$

$$\tau_w = -9\cdot 10^{-4}\frac{1}{0.0319} = 0.028\,Pa$$

If the average C_p at front and at the base are given by $\overline{C_p} = -0.5$ and $\overline{C_p} = -1.5$, respectively using:

$$\overline{C_p} = \frac{1}{S}\int C_p\cdot dS$$

and the flat plate assume an inclination $\alpha_s = 14°$;



6 - Calculate the lift

$$-q_\infty\cdot\int C_{p,b}\cdot n\cdot j\cdot dS = -q_\infty\cdot C_{p,b}\cdot\int\cos\alpha\cdot c\cdot dL_z = -q_\infty\cdot C_{p,b}\cdot\cos\alpha\cdot c$$

$$-q_\infty\int C_{p,f}\cdot n\cdot j\cdot dS = -q_\infty\cdot C_{p,f}\cdot\int -\cos\alpha\cdot c\cdot dL_z = q_\infty\cdot C_{p,f}\cdot\cos\alpha\cdot c$$

$$L = L_f + L_b = q_\infty\cdot C_{p,f}\cdot\cos\alpha\cdot c - q_\infty\cdot C_{p,b}\cdot\cos\alpha\cdot c$$

$$q_\infty = \frac{1}{2}\rho\cdot U_\infty^2 = \frac{1}{2}\cdot 1.2\cdot 50^2 = 1500\,Pa$$

$$L = 1500\cdot(-0.5)\cdot\cos 14\cdot 2 - 1500\cdot(-1.5)\cdot\cos 14\cdot 2$$

$$L = -1455.444 + 4366.331$$

$$L = 2910.887\,N/m$$

7 - Calculate the form drag:

$$-q_\infty\int C_{p,f}\cdot\hat{n}\cdot i\,dS = -q_\infty\cdot C_{p,f}\cdot(-\sin\alpha)\cdot c = q_\infty\cdot C_{p,f}\cdot\sin\alpha\cdot c$$

$$-q_\infty\int C_{p,b}\cdot\hat{n}\cdot j\,dS = -q_\infty\cdot C_{p,b}\cdot\sin\alpha\cdot c = -q_\infty\cdot C_{p,b}\cdot\sin\alpha\cdot c$$

$$D = D_f + D_b = q_\infty\cdot C_{p,f}\cdot c\cdot\sin\alpha - q_\infty\cdot C_{p,b}\cdot c\cdot\sin\alpha$$

$$D = 1500\cdot(-0.5)\cdot 2\cdot\sin 14° - 1500\cdot(-1.5)\cdot 2\cdot\sin 14°$$

$$D = -362.883 + 1088.649$$

$$D = 725.766\,N/m$$

Finally, it is possible to visualize that as soon the flat plate admits an angle, it is created a drag and lift components due to the pressure distribution over the surfaces. Hence it is just to derive the pressure coefficient about the area by the following equations;

$$F_p = -q_\infty\int C_p\cdot\vec{n}\cdot dS$$

$$D' = -q_\infty\int C_p\cdot\vec{n}\cdot i\cdot dS = -q_\infty\cdot C_p\cdot c\cdot\int\vec{n}\cdot i\cdot dL_z \longrightarrow \text{per unit length}$$

$$L' = -q_\infty\int C_p\cdot\vec{n}\cdot j\cdot dS = -q_\infty\cdot C_p\cdot c\cdot\int\vec{n}\cdot j\cdot dL_z \longrightarrow \text{per unit length}$$

However, there are two faces of the flat plate, both are exposed to the pressure distribution, thus;

$$D' = q_\infty\cdot C_p\cdot c\cdot\sin\alpha\int dL_z - q_\infty\cdot C_p\cdot c\cdot\sin\alpha\int dL_z$$

$$L' = q_\infty\cdot C_p\cdot c\cdot\cos\alpha\int dL_z - q_\infty\cdot C_p\cdot c\cdot\cos\alpha\int dL_z$$

the $\sin\alpha$ and $\cos\alpha$ change due to the orientation of vector $\vec{n}$ respective to the vector $\vec{i}$ and j, if pointing to the positive direction, the product $\vec{n}\cdot i$ and $\vec{n}\cdot j$ will be positive. Conversely, when $\vec{n}$ is at opposite direction, the products $\vec{n}\cdot i$ and $\vec{n}\cdot j$ are negative.