Aerodynamic exam 120618

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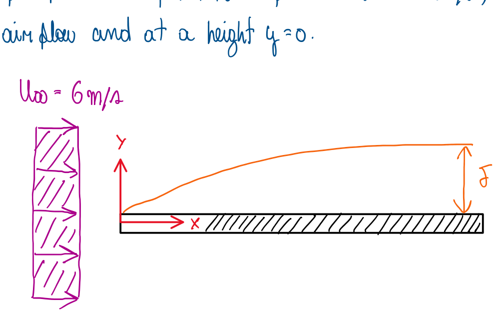
A flat plate is exposed to a flow with $U_\infty = 6 m/s$, its chord is $a = 0.4 m$ and is displaced parallel to the air flow and at a height $y = 0$.



1 - Calculate the total resistance

$$q_\infty = \frac{1}{2}\rho \cdot U_\infty^2 = \frac{1}{2}\cdot 1.2 \cdot 6^2 = 21.6 \text{ Pa}$$

$$Re = \frac{U_\infty \cdot x}{\nu} = \frac{6 \cdot 0.4}{15 \cdot 10^{-6}} = 160000 = 1.6 \cdot 10^5 \rightarrow \text{LAMINAR}$$

$$C_f = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{160000}} = 0.00332 = 3.32 \cdot 10^{-3}$$

$$D_f = C_f \cdot A \cdot q_\infty = 0.00332 \cdot 0.4 \cdot 21.6 = 0.0286848 \text{ N/m}$$

$$2 \text{ faces} \rightarrow 2 \cdot D_f = D_T \rightarrow D_T = 0.0573696 \text{ N/m}$$

As can be seen, the drag resistance of a flat plate is very small, even with the contribution of both surfaces of a flat plate, which is the reason why D_f is multiplied by two. Hence the total drag in a flat plate is contributions of both surfaces.

2 - Assuming that the velocity profile is give by:

$$\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

express the vorticity at the upper surface of the flat plate of the flat plate $\omega(x, y=0)$ as function of x using the velocity. The definition of vorticity is the curl of the velocity gradient, that if developed for the flat plate case results in $\hat{\omega}_z \approx -\partial u/\partial y$, thus it is possible write:

$$\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \rightarrow u = U_\infty\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]$$

$$\frac{\partial u}{\partial y} = U_\infty\left[2\frac{1}{\delta} - 2\frac{y}{\delta}\right]$$

$$\hat{\omega} = -\frac{\partial u}{\partial y} = -U_\infty\left[2\frac{1}{\delta} - 2\frac{y}{\delta}\right] = -2 \cdot \frac{U_\infty}{\delta}$$

$$\delta = 4.91 \cdot \frac{x}{\sqrt{Re}} = 4.91 \cdot \frac{x}{\frac{U_\infty^{1/2} \cdot x^{1/2}}{\nu}} = 4.91 \cdot \frac{\nu^{1/2} \cdot x}{U_\infty^{1/2} \cdot x^{1/2}}$$

$$\delta = 4.91 \cdot \frac{\nu^{1/2} \cdot x^{1/2}}{U_\infty^{1/2}}$$

$$\hat{\omega} = -2 \cdot \frac{U_\infty}{4.91 \frac{\nu^{1/2} \cdot x^{1/2}}{U_\infty^{1/2}}} = -\frac{2}{4.91} \cdot U_\infty \cdot \frac{U_\infty^{1/2}}{\nu^{1/2} \cdot x^{1/2}} = -\frac{2}{4.91} \cdot \frac{U_\infty^{3/2}}{\nu^{1/2} \cdot x^{1/2}}$$

$$\hat{\omega} = -\frac{2}{4.91} \cdot \sqrt{\frac{U_\infty^3}{\nu \cdot x}} = -\frac{2}{4.91} \cdot \sqrt{\frac{6^3}{15 \cdot 10^{-6} \cdot 0.4}} = -2443.992 \frac{m}{s}$$

As can be seen, the vorticity exhibited by an airflow when pass over a flat plate has a very high value. Even though the flow is laminar. This occur, because the vorticity over a flat plate is very sensible to the velocity and the longitudinal length of the plate.

3 - Express the volumetric flow rate in an analytic for as a function of x:

Through a similar method it is possible to calculate the volumetric flow rate This parameter is basically an integration of the velocity profile from zero to the boundary layer thickness.

However, this method can only be assumed if the flow is uniform and aligned with the longitudinal direction, which is the x coordinate. The flow rate due to the displacement thickness is given by:

$$\dot{q} = U_\infty \cdot (\delta - \delta^*)$$

$$\delta^* = \int_0^{y \geq \delta}\left(1 - \frac{u(y)}{U_\infty}\right)dy = \int_0^\delta\left[1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right]dy = \int_0^\delta\left[1 - 2\frac{y}{\delta} + \frac{y^2}{\delta^2}\right]dy$$

$$\delta^* = y\Big|_0^\delta - \frac{2}{\delta}\cdot\frac{1}{2}y^2\Big|_0^\delta + \frac{1}{\delta^2}\cdot\frac{1}{3}\cdot y^3\Big|_0^\delta = \delta - \frac{1}{\delta}\cdot\delta^2 + \frac{1}{3}\cdot\frac{1}{\delta^2}\cdot\delta^3$$

$$\delta^* = \delta - \delta + \frac{1}{3}\cdot\delta = \frac{1}{3}\delta$$

$$\dot{q} = U_\infty(\delta - \delta^*) = U_\infty\left(\delta - \frac{1}{3}\delta\right) = U_\infty\left(\frac{3\delta - \delta}{3}\right) = U_\infty \cdot \frac{2\delta}{3}$$

$$\dot{q} = U_\infty \cdot \delta \cdot \frac{2}{3} = U_\infty \cdot 4.91 \cdot \frac{x}{\sqrt{Re_x}} \cdot \frac{2}{3} = 4.91 \cdot \frac{2}{3} \cdot \frac{U_\infty \cdot x}{\left(\frac{U_\infty \cdot x}{\nu}\right)^{1/2}}$$

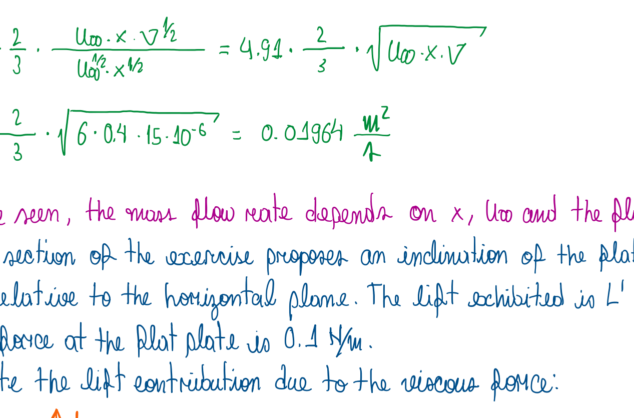
$$\dot{q} = 4.91 \cdot \frac{2}{3} \cdot \frac{U_\infty \cdot x \cdot \nu^{1/2}}{U_\infty^{1/2} \cdot x^{1/2}} = 4.91 \cdot \frac{2}{3} \cdot \sqrt{U_\infty \cdot x \cdot \nu}$$

$$\dot{q} = 4.91 \cdot \frac{2}{3} \cdot \sqrt{6 \cdot 0.4 \cdot 15 \cdot 10^{-6}} = 0.01964 \frac{m^2}{s}$$

As can be seen, the mass flow rate depends on x, U_∞ and the fluid property.

The next section of the exercise proposes an inclination of the flat plate, more precisely $\alpha = 0.1$ rad relative to the horizontal plane. The lift exhibited is $L' = 5.0$ N/m and the absolute viscous force at the flat plate is 0.1 N/m.

4 - Calculate the lift contribution due to the viscous force:



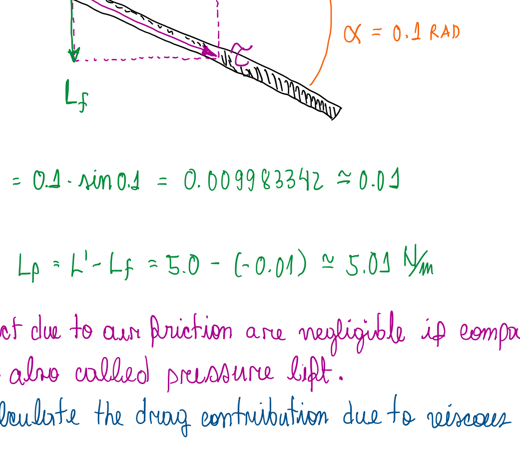
$$L_f = \tau \cdot \sin(\alpha) = 0.1 \cdot \sin 0.1 = 0.009983342 \simeq 0.01$$

$$L_f = -0.01 \text{ N/m}$$

$$L' = L_p + L_f \rightarrow L_p = L' - L_f = 5.0 - (-0.01) \simeq 5.01 \text{ N/m}$$

The effect due to air friction are negligible if compared with the effects due to form lift, also called pressure lift.

5 - Calculate the drag contribution due to viscous effect and form:



$$\sin\alpha = \frac{D_p}{F_p}, \quad \cos\alpha = \frac{L_p}{F_p}; \quad F_p = \frac{D_p}{\sin\alpha} = \frac{L_p}{\cos\alpha}$$

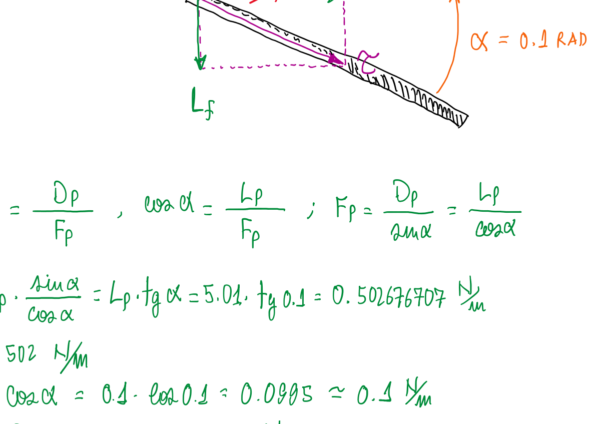
$$D_p = L_p \cdot \frac{\sin\alpha}{\cos\alpha} = L_p \cdot tg\,\alpha = 5.01 \cdot tg\,0.1 = 0.502676707 \text{ N/m}$$

$$D_p = 0.502 \text{ N/m}$$

$$D_f = \tau \cdot \cos\alpha = 0.1 \cdot \cos 0.1 = 0.0995 \simeq 0.1 \text{ N/m}$$

$$D = D_f + D_p = 0.1 + 0.502 = 0.602 \text{ N/m}$$

This calculation is important to understand the components produced by a wing or flat plate. These are described below:

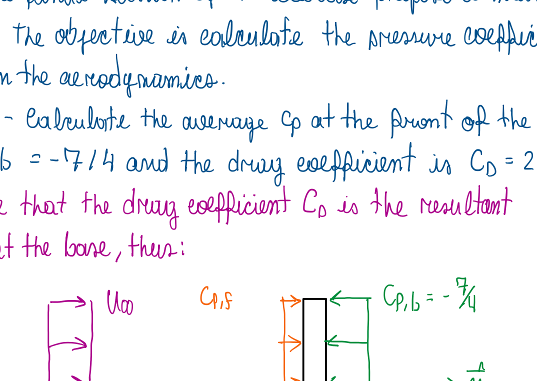


$$L_f = \tau \cdot \sin\alpha$$
$$D_f = \tau \cdot \cos\alpha$$
$$L_p = F_p \cdot \cos\alpha$$
$$D_p = F_p \cdot \sin\alpha$$

Hence, it is possible to notice that the forces acting on a wing is all a matter of geometry and trigonometry.

The final section of the exercise propose a massive slope for the flat plate $\alpha = \pi/2$ RAD. The objective is calculate the pressure coefficient and understand the impact of the shape on the aerodynamics.

6 - Calculate the average Cp at the front of the flat plate considering that its base $C_{p,b} = -7/4$ and the drag coefficient is $C_D = 2.0$: In these cases, a plate at 90° can assume that the drag coefficient C_D is the resultant between pressure coefficient at the front and at the base, thus:



$$-q_\infty \cdot \int C_p \cdot n \cdot i \cdot ds$$

$$-q_\infty \int C_{p,b} \cdot 1 \cdot c \cdot dL_z \rightarrow D_b = -q_\infty \cdot C_{p,b} \cdot c \int dL_z = -q_\infty \cdot C_{p,b} \cdot c$$

$$D_b = -q_\infty \cdot C_{p,b} \cdot c$$

$$-q_\infty \int C_{p,f} \cdot (-1) \cdot c \cdot dL_z \rightarrow D_f = q_\infty \cdot C_{p,f} \cdot c$$

$$D = D_F + D_B = q_\infty \cdot C_D \cdot c \rightarrow q_\infty \cdot C_D \cdot c = q_\infty \cdot C_{p,f} \cdot c + (-q_\infty \cdot C_{p,b} \cdot c)$$

$$C_D = C_{p,f} - C_{p,b} \rightarrow C_{p,f} = C_D + C_{p,b}$$

$$C_{p,f} = 2.0 - \frac{7}{4} = \frac{8-7}{4} = \frac{1}{4} = 0.25$$

7 - Considering the frontal Cp is assymetrically distributed and this is described by the following equation:

$$C_p \approx \left(1 - \frac{9y^2}{a^2} - \frac{y}{a}\right)$$

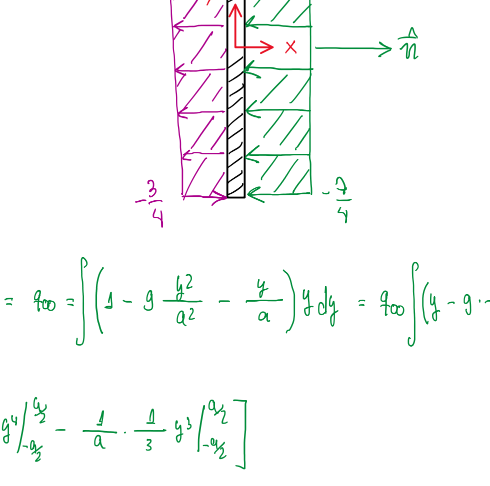
Calculate the torque about the z axis in $(x = 0, y = 0)$:

$$C_{p,f}(a/2) = \left(1 - \frac{9\left(\frac{a}{2}\right)^2}{a^2} - \frac{\frac{a}{2}}{a}\right) = 1 - \frac{9a^2}{4}\cdot\frac{1}{a^2} - \frac{a}{2}\cdot\frac{1}{a}$$

$$C_{p,f}\left(\frac{a}{2}\right) = 1 - \frac{9}{4} - \frac{1}{2} = \frac{1}{2} - \frac{9}{4} = \frac{4-18}{8} = -\frac{14}{8} = -\frac{7}{4}$$

$$C_{p,f}(-a/2) = 1 - \frac{9\left(-\frac{a}{2}\right)^2}{a^2} - \frac{\left(-\frac{a}{2}\right)}{a} = 1 - \frac{9a^2}{4}\cdot\frac{1}{a^2} + \frac{a}{2}\cdot\frac{1}{a}$$

$$C_{p,f}\left(-\frac{a}{2}\right) = 1 - \frac{9}{4} + \frac{1}{2} = \frac{3}{2} - \frac{9}{4} = \frac{12-18}{8} = -\frac{6}{8} = -\frac{3}{4}$$



$$M_z = \int_{-a/2}^{a/2} q_\infty \cdot C_{p,f} \cdot y\, dy = q_\infty = \int\left(1 - 9\frac{y^2}{a^2} - \frac{y}{a}\right)y\,dy = q_\infty\int\left(y - 9\cdot\frac{y^3}{a^2} - \frac{y^2}{a}\right)dy$$

$$M_z = q_\infty\left[\frac{1}{2}y^2\Big|_{-a/2}^{a/2} - \frac{9}{a^2}\frac{1}{4}y^4\Big|_{-a/2}^{a/2} - \frac{1}{a}\cdot\frac{1}{3}y^3\Big|_{-a/2}^{a/2}\right]$$

$$M_z = q_\infty\left[\frac{1}{2}\left(\frac{a^2}{4} - \frac{a^2}{4}\right)^0 - \frac{9}{4a^2}\left(\frac{a^4}{16} - \frac{a^4}{16}\right)^0 - \frac{1}{3a}\cdot\left(\frac{a^3}{8} + \frac{a^3}{8}\right)\right]$$

$$M_z = -q_\infty\frac{1}{3a}\cdot\frac{2a^3}{8} = -q_\infty \cdot a^2 \cdot \frac{1}{12}$$